

PTCC: NEW BEAM DYNAMICS DESIGN CODE FOR LINEAR ACCELERATORS*

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Abstract

A fast and accurate beam dynamics design code, named PTCC (Particle Tracking Code in C) is developed to simulate particles dynamic in linear accelerators. PTCC solves the relativistic equations of motion for the macro-particles when subjected to electromagnetic fields excited in RF cavities. The self-fields of the particles are also part of the electromagnetic fields through which the particles are tracked. Self-fields are calculated using a modified 2D cylindrically symmetric mesh based method, making use of beam and field symmetry to provide fast simulation. The code has been benchmarked with the well known code ASTRA which is used mainly in simulations of next generation FEL linacs. PTCC provides a new tool for designing buncher section of linear accelerators that convert DC beam into bunches. New buncher design tool and benchmark results of PTCC with ASTRA are presented.

INTRODUCTION

Simulation of charged particle dynamics in accelerators is essential for the design and understanding of these machines. There are many of particle tracking codes capable of simulating linear accelerators (e.g. PARMELA [1], GPT [2] and ASTRA [3]). However, these codes are not open for modifications or customization, e.g. changing meshing, for a certain application to adapt to experiment configuration. Hence, a new code is needed to facilitate design and simulation of different parts of linear accelerators.

EQUATIONS OF MOTION

The best choice for coordinates system for accelerator code is cylindrical coordinates because of field and shape symmetry of accelerating cavities and accelerator components. Problem of using cylindrical coordinates is it has singularity at $r = 0$. The singularity can be handled by using complex coordinates [4]. The Lorentz equations of motion in Cartesian coordinates are given by:

$$\frac{d}{dt}(\gamma\beta_x) = \frac{q}{m_0c_0}(E_x + c_0\beta_y B_z - c_0\beta_z B_y) \quad (1)$$

$$\frac{d}{dt}(\gamma\beta_y) = \frac{q}{m_0c_0}(E_y + c_0\beta_z B_x - c_0\beta_x B_z) \quad (2)$$

$$\frac{d}{dt}(\gamma\beta_z) = \frac{q}{m_0c_0}(E_z + c_0\beta_x B_y - c_0\beta_y B_x) \quad (3)$$

For a vector A with polar position (r, θ) , transformation from Cartesian coordinates is given by:

$$\begin{aligned} A_x &= A_r \cos(\theta) - A_\theta \sin(\theta) \\ A_y &= A_r \sin(\theta) + A_\theta \cos(\theta) \end{aligned} \quad (4)$$

From those equations, the following relations can be easily derived:

$$\begin{aligned} A_x + iA_y &= (A_r + iA_\theta)e^{i\theta} \\ A_x - iA_y &= (A_r - iA_\theta)e^{-i\theta} \end{aligned} \quad (5)$$

Introducing complex velocity variable $\beta_c = \beta_x + i\beta_y$ and complex position $R_c = x + iy = re^{i\theta}$, then by adding Eq. (1) and Eq. (2) multiplied by i :

$$\begin{aligned} \frac{d}{dt}(\gamma(\beta_x + i\beta_y)) &= \frac{q}{m_0c_0}((E_x + iE_y) \\ &\quad - ic_0(v_x + iv_y)B_z + ic_0v_z(B_x + iB_y)) \\ \frac{d}{dt}(\gamma\beta_z) &= \frac{q}{m_0c_0} \\ &\quad (E_z - Im\{c_0(v_x + iv_y)(B_x - iB_y)\}) \end{aligned} \quad (6)$$

Using Eq. (5), Eq. (6) can be written in terms of cylindrical coordinates by:

$$\begin{aligned} \frac{d}{dt}(\gamma\beta_c) &= \frac{q}{m_0c_0}((E_r + iE_\theta)e^{i\theta} - ic_0\beta_c B_z \\ &\quad + ic_0\beta_z(B_r + iB_\theta)e^{i\theta}) \\ \frac{d}{dt}(\gamma\beta_z) &= \frac{q}{m_0c_0}(E_z + Im\{c_0\beta_c(B_r - iB_\theta)e^{-i\theta}\}) \\ \frac{dR_c}{dt} &= c_0\beta_c, \quad \frac{dz}{dt} = c_0\beta_z \end{aligned} \quad (7)$$

Those equations uses the cylindrical fields components without introducing any singularities. The equations of motion are solved using 4th order Runge Kutta (RK4) [5] solver with fixed time step for $\gamma\beta_c$ and $\gamma\beta_z$. The fields \vec{E} and \vec{B} in equation of motion composed of two parts: the external accelerating fields \vec{E}_{ex} , \vec{B}_{ex} and the beam self fields (Space Charge) \vec{E}_s , \vec{B}_s . Both field types are determined on the cylindrical grid but calculated and entered by different ways. External fields are obtained using an external field program in the form of a list at specific points on a 2D grid points (r, z) . PTCC uses fields (electromagnetic for cavities and magneto-static for Solenoids) exported from field solver of Los Alamos POSSION/SUPERFISH suite [6]. Although Eq. (7) could be used to track particles over the beam cross section with different initial azimuthal position θ , due to cylindrical symmetry of the fields, it is sufficient to consider particles at one initial θ , i.e. $\theta = 0$.

* Work supported by the Egyptian Science and Technology Development Fund (STDF) No. 953.

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SPACE CHARGE FIELDS

The spacecharge fields (self fields) have to be computed at each time step of the numerical integration of the relativistic equation of motion. The spacecharge calculation is performed similar to space charge model of PARMELA code as follows:

- Transformation of the particles from the laboratory frame to the rest frame by Lorentz transformation.
- Beam area is divided into r, z meshes in rest frame.
- Particles then are assigned to grid rings using Cloud-In-Cell (CIC) model [7].

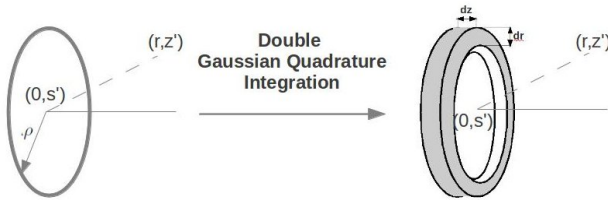


Figure 1: Ring and quadrature integration.

- Electric field at point $(r' = r, z')$ from a charged ring at longitudinal position s' with radius ρ and uniform charge density λ in rest frame (Fig. 1) is given by [8],

$$E'_r = \frac{\rho\lambda}{2\pi\epsilon_0 r d^{\frac{3}{2}}} \left(K(\alpha) - \frac{\rho^2 + (z' - s')^2 - r^2}{d - 4\rho r} E(\alpha) \right) \quad (8)$$

$$E'_z = \frac{\rho\lambda}{\pi\epsilon_0} \frac{(z' - s')E(\alpha)}{d^{\frac{3}{2}}(d - 4\rho r)}, \quad (9)$$

$$d = r^2 + \rho^2 + (z' - s')^2 + 2\rho r$$

where ρ is ring radius and $(z' - s') = \gamma(z - s)$ is the longitudinal distance between ring at s and observing point (r, z) in ring rest frame, $\alpha = \sqrt{4\rho r / (r^2 + \rho^2 + (z' - s')^2 + 2\rho r)}$ and $K(\alpha)$ and $E(\alpha)$ are complete elliptical integrals of the first and second kinds, respectively [9].

- The fields from ring of charge of cell $drdz$ at point (r, z') can be obtained from Eq. (9) and Eq. (8) by double integration in the ρ - s' space as

$$E'_z(r, z') = \int_{r_k}^{r_{k+1}} \int_{z_1}^{z_2} E'_z(r, z') d\rho ds' \quad (10)$$

$$E'_r(r, z') = \int_{r_k}^{r_{k+1}} \int_{z_1}^{z_2} E'_r(r, z') d\rho ds' \quad (11)$$

These two integrals are evaluated using numerical integration using Gaussian Quadrature Integration [10].

- The field contributions of the individual rings are added up at grid centers. Then, the fields in the laboratory frame are obtained using the Lorentz field transformation from rest to moving frame [11],

$$\vec{E} = \gamma \vec{E}' - \frac{\gamma^2}{\gamma + 1} (\vec{\beta} \cdot \vec{E}') \vec{\beta}, \quad \vec{B} = \gamma \frac{\vec{\beta}}{c} \times \vec{E}' \quad (12)$$

- Space charge fields at particles positions are obtained by simple linear interpolation from grid center points calculated fields.

BUNCHER TOOL

The most complex design part in linac is the buncher section, which is responsible for accepting most of the beam coming from DC gun, converting it into bunches and accelerating those bunches to relativistic energy. To facilitate the design of buncher section of Linac, PTCC provides a buncher tool used to analysis and optimize beam dynamics in the buncher section. PTCC buncher is a 1D design tool used to track on-axis particle (position, momentum and energy) in on-axis longitudinal electric field (1D) only using 4th order Runge-Kutta integrator without space charge fields (on-axis at $r = 0$ no radial electric field component exist (external or space charge) and longitudinal space charge field is neglected compared to the applied RF longitudinal field). By changing the buncher cavities dimensions and using the on-axis field exported from SUPERFISH, the buncher can be designed.

BENCHMARK OF PTCC

To benchmark PTCC with ASTRA, FNPL (Fermilab NICADD Photoinjector Laboratory) photo-injector Linac [12] is simulated using both ATSRA and PTCC. FNPL accelerator consists of a 1+1/2 cell L-band ($f = 1.3$ GHz) RF-gun equipped with a high quantum efficiency Cesium-Telluride photocathode allowing the photoemission of electron bunches with charge up to approximately 15 nC. The generated bunches are further accelerated, up to 16 MeV, by a downstream TESLA-type 9-cells superconducting accelerating cavity operating with an accelerating gradient of approximately 25 MV/m and operating frequency of 1.3 GHz. Downstream of the TESLA cavity the beam line includes a set of quadrupoles and steering dipoles elements for beam focusing and orbit correction. The beamline also incorporates a magnetic bunch compressor (chicane) which enhances the bunch peak current up to 2.5 kA.

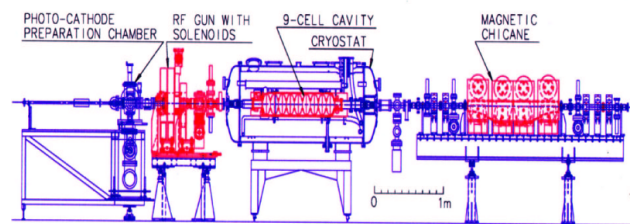
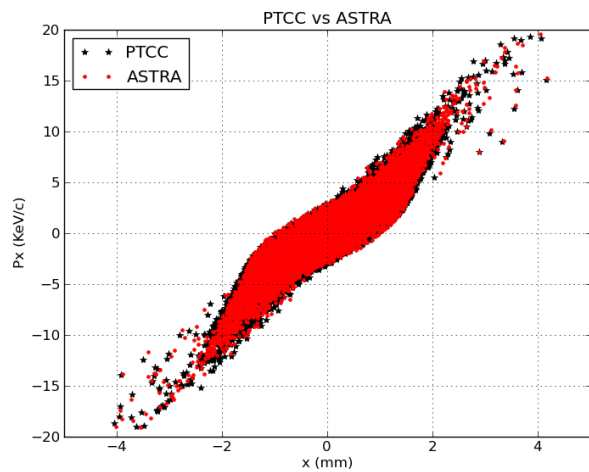
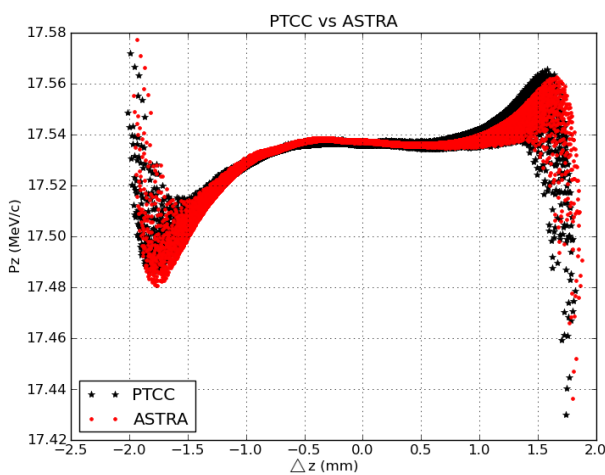


Figure 2: FNPL accelerator layout.

The longitudinal and transverse phase space from PTCC is been compared to results from ASTRA simulation till point before chicane (Figs. 3, 4). Figures 5, 6 show comparison of different beam parameters from ASTRA and PTCC at linac output.

Figure 3: Transverse phase space at $z = 3.77$ m.Figure 4: Longitudinal phase space at $z = 3.77$ m.

CONCLUSION

A new code that can be used for the analysis and the design of linear accelerator is presented. The code simulates the 2D motion of particles along the different parts of the linac. A simplified version of that code, considering only 1D axial motion in absence of space charge, can be employed in designing the buncher section of a linac system.

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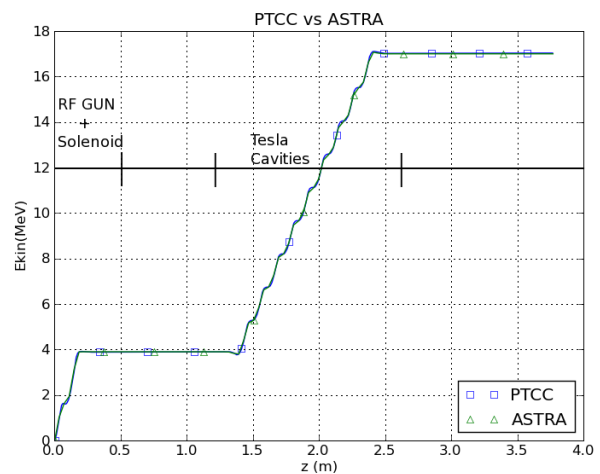


Figure 5: Average beam energy through linac beam line.

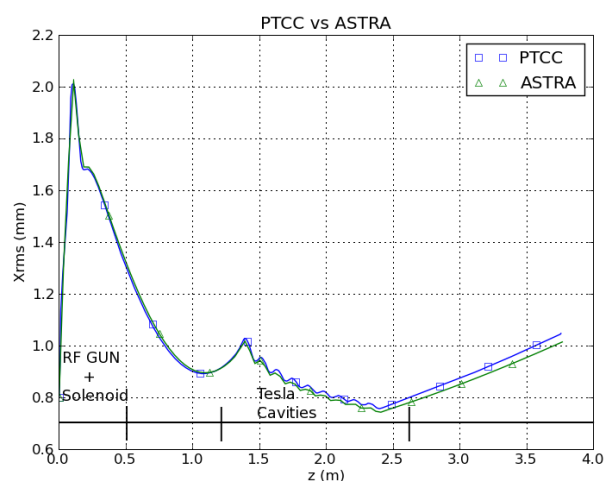


Figure 6: Transverse beam size through linac beam line.

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